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A SIMPLE FRACTURE MECHANICS BASED METHOD FOR FATIGUE LIFE PREDICTION IN THICK-WALLED CYLINDERS

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A method is proposed for predicting the fatigue life of thick-walled cylinders based on numerical integration of the fatigue crack growth curve as determined from a fracture mechanics analysis. The effects of autofrettage residual stresses, crack shape, and of the compressive portion of the stress intensity factor are accounted for. A method for correcting the autofrettage residual stresses for the Bauschinger Effect, based on recent analytical results by

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20. ABSTRACT (CONT'D)

Peter Chen, is also included. A simple computer program for performing the calculation of fatigue life is presented along with a comparison of the calculated results with the experimental results of Davidson, and of Throop and Fujczak.

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ACKNOWLEDGEMENTS

This paper is dedicated to the memory of the late Joseph Throop whose pioneering work in this area formed the basis of the work reported herein as well as much of the other work cited. Helpful discussions with J. H. Underwood and A. P. Parker are also acknowledged.

NOMENCLATURE

a	-	crack depth
a_o	-	initial crack depth
N	-	number of fatigue cycles
C,m	-	material constants in crack growth equation
K	-	stress intensity factor
K_c	-	critical value of K
K_{max}	-	maximum value of K during fatigue cycle
KP	-	K due to pressure
KR	-	K due to residual stress
ΔK	-	range of K during fatigue cycle
σ_t	-	tangential stress
σ_{tR}	-	residual σ_t
SY	-	yield stress
R	-	radius
RO	-	outer radius
RI	-	inner radius
RP	-	radius of elastic-plastic interface
R_a	-	radius at crack tip
SA	-	σ_{tR} at $R = R_a$
SB	-	σ_{tR} at R = RI corrected for Bauschinger Effect
RZ	-	radius at which $\sigma_{tR} = 0$
SF	-	crack shape factor
F	-	portion of negative KR included in ΔK
Y	-	diameter ratio, RO/RI

INTRODUCTION

In many cases, the fatigue life of a structural component can be predicted with reasonable accuracy by determining the maximum range of operating cyclic stress intensity; and using this with stress vs. cycles (S/N) curve, obtained from an axial fatigue test on small specimens, determine fatigue life. Fatigue lives predicted by this method for thick-walled cylinders have generally not been in reasonable agreement with the experimental results (refs 1,2). This is particularly true if the cylinders contain residual stresses produced by autofrettage.

This disagreement is due to the fact that a large portion of the fatigue life of a thick-walled cylinder may be consumed by propagation of the fatigue crack. Another reason is the fact that the internal pressure acts directly on the surfaces of microscopic voids and defects on the bore surface which may decrease the number of cycles for crack initiation and early growth.

Based on the above, it is proposed that a more accurate prediction of fatigue lives of pressurized, thick-walled cylinders can be obtained from a fatigue crack propagation model rather than a model based on the S/N curve approach. This has the added advantage of being able to readily account for the presence of residual stresses due to autofrettage or shrink fitting or other processes.

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[1]Eisenstadt, R., Kendall, D. P., and Davidson, T. E., "A Comparison of Results of Axial-Tension, Rotating-Beam, and Pressurized Cylinder Fatigue Tests," Experimental Mechanics, Vol. 9, No. 6, June 1969, pp. 250-254.

[2]Karl, E., "The Fatigue Strength of Thick-Walled Cylinders," Proceedings of the 9th AIRAPT High Pressure Conference, Elsevier, 1984.

THEORY

It has been shown that the rate of propagation of a fatigue crack can be determined from the applied range of stress intensity factor, (ΔK), associated with the crack and the cyclic loading applied during the fatigue cycle, by the following equation.

$$da/dN = C(\Delta K)^m \tag{1}$$

where a is the crack depth, N the number of fatigue cycles, and C and m are material constants.

The fatigue life of a thick-walled cylinder can therefore be predicted by integration of Eq. (1) if the following assumptions are made:

a. The constants C and m are known for the cylinder material.

b. All materials contain crack-like flaws which can be assumed to be initial cracks from which fatigue cracks will propagate.

c. Failure of the cylinder will occur when, or very soon after, the maximum value of K exceeds some critical value, K_c.

d. The range of stress intensity factor during the fatigue cycle can be calculated as a function of pressure, crack size, and residual stress distribution.

Equation (1) can then be integrated from an assumed initial crack depth to the crack depth at which $K_{max} = K_c$, to determine the fatigue life.

RESIDUAL STRESS DISTRIBUTION

In calculating the effect of residual stress on stress intensity factor, only the tangential component need to be considered since the longitudinal and radial stress components do not contribute to the K value for a crack, whose plane is normal to the tangential direction.

Based on the Tresca yield criteria and assuming ideal elastic-perfectly-plastic material behavior, the tangential residual stress distribution in an autofrettaged thick-walled cylinder is given by (ref 3):

$$\frac{\sigma_{tR}}{SY} = \frac{RO^2 + RP^2}{2(RO)^2} + \ln\left(\frac{R}{RP}\right) - \frac{RI^2}{RO^2 - RI^2}\left(\frac{RO^2 - RP^2}{2RO^2} + \ln\frac{RP}{RI}\right)\left(1 + \frac{RO^2}{R^2}\right) \qquad (2)$$

Equation (2) is based on the assumption that the material behaves elastically on the release of the autofrettage pressure. However, most materials, particularly the quenched and tempered, low alloy steels, generally used for high pressure vessels, exhibit a significant Bauschinger effect. This is a reduction in the compressive yield strength resulting from prior plastic deformation in tension. Thus the material near the bore surface generally does not behave elastically during unloading, but actually "reverse yields" in compression which significantly reduces the residual tangential stresses near the bore from those calculated from Eq. (2). The effects of this phenomenon on the final residual stress distribution has been considered by several investigators (refs 4-7). A general analytical approach to the

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[3]Timoshenko, S., Strength of Materials, Part II, Van Nostrand, 1956, pp. 386-392.

[4]Franklin, G. J. and Morrison, J. L. M., "Autofrettage of Cylinders: Prediction of Pressure/External Expansion Curves and Calculation of Residual Stresses," Institution of Mechanical Engineers, London, Vol. 174, 1960, p. 947.

[5]Sidebottom, O. M., Chu, S. C., and Lamba, H. S., "Unloading of Thick-Walled Cylinders That Have Been Plastically Deformed," Experimental Mechanics, Vol. 16, No. 12, December 1976, pp. 454-460.

[6]Milligan, R. V., Koo, W. H., and Davidson, T. E., "The Bauschinger Effect in a High Strength Steel," ASME Publication No. 65-MET-9, 1965.

[7]Kendall, D. P., "The Effect of Material Removal on the Strength of Autofrettaged Cylinders," Watervliet Arsenal Technical Report WVT-7003, January 1970 (available from NTIS).

calculation of resulting residual stresses based on realistic modelling of the actual material behavior has been proposed by Chen (ref 8).

From an evaluation of Chen's results, along with the experimental results of References 4 through 7, the following general conclusions regarding the actual distribution of residual tangential stress can be reached.

a. If the absolute value of residual stress at the bore, calculated from Eq. (2), does not exceed 0.4 times the original yield strength, results of Eq. (2) can be used with no correction.

b. The distribution of residual tangential stress can be assumed to be linear between the bore radius and the radius at which Eq. (2) gives a value of $\sigma_{tR} = 0$.

c. The actual residual stresses at the bore are significantly less than those that would be predicted from Eq. (2). However, due to the strain-hardening that is associated with the reduced compressive yield strength, the actual residual stresses are related to the calculated values. In other words, the more the calculated residual stress exceeds the reduced compressive

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[4]Franklin, G. J. and Morrison, J. L. M., "Autofrettage of Cylinders: Prediction of Pressure/External Expansion Curves and Calculation of Residual Stresses," Institution of Mechanical Engineers, London, Vol. 174, 1960, p. 947.

[5]Sidebottom, O. M., Chu, S. C., and Lamba, H. S., "Unloading of Thick-Walled Cylinders That Have Been Plastically Deformed," Experimental Mechanics, Vol. 16, No. 12, December 1976, pp. 454-460.

[6]Milligan, R. V., Koo, W. H., and Davidson, T. E., "The Bauschinger Effect in a High Strength Steel," ASME Publication No. 65-MET-9, 1965.

[7]Kendall, D. P., "The Effect of Material Removal on the Strength of Autofrettaged Cylinders," Watervliet Arsenal Technical Report WVT-7003, January 1970 (available from NTIS).

[8]Chen, P. C. T., "Prediction of Residual Stresses in an Autofrettaged Cylinder," Proceedings of the 9th AIRAPT High Pressure Conference, Elsevier, 1984.

yield strength, the more reverse yielding or plastic strain will occur on unloading. This will result in an increase in compressive yield strength due to strain-hardening.

Based on the above observations, the following equation is proposed for the residual tangential stress at the bore, corrected for the Bauschinger effect, defined as (SB).

$$\begin{aligned} SB &= \sigma_{tRI} \quad \text{for } \sigma_{tRI} > -0.4\ SY \\ SB &= -0.4(SY) + 0.5\,[\sigma_{tRI} + 0.4(SY)] \quad \text{for } \sigma_{tRI} < -0.4\ SY \end{aligned} \tag{3}$$

where σ_{tRI} is the value of σ_{tR} at R = RI from Eq. (2). The residual bore stresses from Eq. (3) are compared with those from Chen's analysis (ref 8) in Figure 1.

Defining RZ as the radius at which the value of σ_{tR} calculated from Eq. (2) is zero, and assuming a linear distribution of σ_{tR} between RI and RZ results in the following equation:

$$\sigma_{tR} = SB\left(\frac{RZ - R}{RZ - RI}\right) \tag{4}$$

STRESS INTENSITY FACTORS

A number of investigators have published solutions for the stress intensity factor associated with a crack in a pressurized thick-walled

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[8]Chen, P. C. T., "Prediction of Residual Stresses in an Autofrettaged Cylinder," Proceedings of the 9th AIRAPT High Pressure Conference, Elsevier, 1984.

cylinder (refs 9-12). Much of this work is related to the effects of residual stresses, crack shape, and multiple cracks, all of which significantly affect the value of K. A good summary of this work can be found in Parker, et al (ref 13). This reference also gives one of the most accurate solutions available for the effect of autofrettage residual stresses on straight-fronted, single, internal cracks. Underwood and Throop (ref 14) previously published an approximate solution assuming a linear varying residual stress over the crack depth. For the case given as an example in Parker, et al (ref 13), the values of K calculated using the Underwood and Throop solution agree with the more rigorous analysis within a few percent.

Underwood and Throop (ref 14) give the following equations for the stress intensity factors due to both the internal pressure (KP) and to the autofrettage residual stress (KR)

$$KP = \left[\frac{2.24\ PY^2}{Y^2 - 1}\right](SF)\sqrt{\pi a} \tag{5}$$

$$KR = [1.12\ (SB) - 0.68\ (SB - SA)](SF)\sqrt{\pi a} \tag{6}$$

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[9] Bowie, O. L. and Freese, C. E., "Elastic Analysis for a Radial Crack in a Circular Ring," Engineering Fracture Mechanics, Vol. 4, No. 2, 1972, pp. 315-321.

[10] Pu, S. L. and Hussain, M. A., "Stress Intensity Factors For Radial Cracks in a Partially Autofrettaged Thick-Walled Cylinder," ASTM STP 791, 1983, pp. I-194-I-213.

[11] Tan, C. L. and Fenner, R. T., "Stress Intensity Factors For Semi-Elliptical Cracks in Pressurized Cylinders Using Boundary Integral Equation Method," Int. Jour. of Fracture, Vol. 16, No. 3, 1980, pp. 233-245.

[12] Atulri, N. and Kathiresan, K., "3-D Analysis of Surface Flaws in a Thick-Walled Reactor Pressure Vessel Using Displacement-Hybrid Finite Element Method," Nuclear Engineering and Design, Vol. 51, pp. 163-176.

[13] Parker, A. P., Underwood, J. H., Throop, J. F., and Andrasic, C. P., "Stress Intensity and Fatigue Crack Growth in a Pressurized, Autofrettaged Thick Cylinder," ASTM STP 791, pp. I-216-I-237.

[14] Underwood, J. H. and Throop, J. F., "Surface Crack K-Estimates and Fatigue Life Calculations in Cannon Tubes," ASTM STP 687, 1979, pp. 195-210.

where P = internal pressure

Y = diameter ratio

SB = residual tangential stress at bore

SA = residual tangential stress at the radius located at the crack tip

a = crack depth

SF = crack shape factor (to be discussed later)

CRACK SHAPE

The equations for the stress intensity factors (Eqs. (5) and (6)), are for a straight-fronted crack (constant depth over the whole length of the cylinder). In practice cracks at the bore of a pressurized cylinder usually grow in a semi-elliptical shape. This shape will vary from a semi-circular shape (depth-to-length ratio of about 0.5) to a fairly long semi-ellipse with a depth-to-length ratio of 0.1. Due to the support provided by the uncracked material beyond the ends of the crack, there is a significant decrease in the stress intensity factor with increasing values of depth-to-length ratio.

A number of investigators have developed solutions for crack shape factors for cracked plates, notably that of Newman and Raju (ref 15). These have been used by Parker, et al and others, to estimate the reduction in stress intensity factor due to crack shape. Tan and Fenner (ref 11) and Atulri and Kathiresan (ref 12) have obtained shape factors for specific cases

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[11]Tan, C. L. and Fenner, R. T., "Stress Intensity Factors For Semi-Elliptical Cracks in Pressurized Cylinders Using Boundary Integral Equation Method," Int. Jour. of Fracture, Vol. 16, No. 3, 1980, pp. 233-245.

[12]Atulri, N. and Kathiresan, K., "3-D Analysis of Surface Flaws in a Thick-Walled Reactor Pressure Vessel Using Displacement-Hybrid Finite Element Method," Nuclear Engineering and Design, Vol. 51, pp. 163-176.

[15]Newman, J. C. and Raju, I. S., "Analysis of Surface Cracks in Finite Plates Under Tension and Bending Loads," NASA, TN1578, 1979.

of cracked thick-walled cylinders and have shown less reduction in K due to crack shape than is estimated from the plate solutions, particularly for deep cracks. These results are discussed in detail by Parker, et al (ref 13), and show the complex way in which shape factors vary with crack depth and shape.

Only relatively shallow cracks need be considered, since most of the fatigue life is in the shallow crack portion of the crack growth. Most fatigue cracks in thick-walled cylinders have been found to have a depth-to-length ratio between 0.4 and 0.3, unless some unusual surface damage or defects force a lower ratio. In order to obtain a simple method of calculating fatigue life, it is proposed that a constant value of shape factor of 0.7 be used unless there is reason to expect unusually long initial cracks. In this case, a value of 0.8 or 0.9 can be used for added safety. The value of 0.7 is probably conservative since actual values may be lower than this in many cases. This will be further discussed under Numerical Results. This shape factor is then multiplied times the value of K as shown in Eqs. (5) and (6).

STRESS INTENSITY RANGE

The crack growth rate, as calculated from Eq. (1), is a function of the range of stress intensity factor or the difference between the maximum and minimum value of K during the fatigue cycle (ΔK). If the minimum value of K is zero or greater than zero, there is no problem in defining ΔK. However, in

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[13] Parker, A. P., Underwood, J. H., Throop, J. F., and Andrasic, C. P., "Stress Intensity and Fatigue Crack Growth in a Pressurized, Autofrettaged Thick Cylinder," ASTM STP 791, pp. I-216-I-237.

an autofrettaged cylinder, the value of K calculated from Eq. (6) is negative for shallow cracks when P equals zero. It is generally assumed that when K equals zero the surfaces of the fatigue crack are in contact. If a compressive stress is applied, the crack surfaces can not close any further and thus the K can not become negative. It is common practice to consider that the negative portion of the K range has no effect on crack growth. Therefore, if the minimum K is negative, it is assumed that ΔK equals the maximum value of K. This assumption is sufficiently accurate for fatigue life prediction if the absolute value of the calculated negative K is less than, or of the order of, the maximum K. For autofrettaged cylinders operating at relatively low pressures and for shallow cracks, the calculated absolute value of the negative K at zero pressure is three to five times the maximum K at pressure. Such conditions would be referred to as a load ratio, R (ratio of minimum to maximum stress or K) of -3 to -5. Although there is no known fatigue crack propagation data available for such highly negative R ratios, an examination of fatigue data on autofrettaged cylinders clearly indicates that, under such conditions, the effect of the negative portion of the K range can not be completely neglected in determining an effective ΔK.

As previously discussed, the maximum K is the algebraic sum of the K due to pressure (KP) and the K due to the residual stresses (KR).

$$Kmax = KP + KR$$

Since, for shallow cracks KR is negative and if the negative portion of the K range is neglected, then

$$\Delta K = K_{max}$$

If some fraction (F) of the negative portion of the K range is considered to be part of the K range, then

$$\Delta K = K_{max} - (F)KR \qquad (KR < 0)$$

or

$$\Delta K = KP + KR - (F)KR$$

$$= KP + (1 - F)KR \tag{7}$$

RESULTS

A final equation for ΔK as a function of pressure, residual stress, and crack shape can now be written by combining Eqs. (4) through (7).

$$\Delta K = (SF)\sqrt{\pi a}\{\frac{2.24\ PY^2}{Y^2 - 1} + (SB)[1.12 - .68(\frac{Ra - RI}{RZ - RI})](1-F)\} \tag{8}$$

where

$$SB = -0.4(SY) + 0.5[\sigma_{tRI} + 0.4(SY)] \tag{3}$$

The total fatigue life can be determined by integrating Eq. (1) between the limits of the initial crack depth, a_o, and a final value of "a" at which K equals a critical value. This could be accomplished by substituting ΔK from Eq. (8) into Eq. (1) and performing a closed form integration between the appropriate limits. Since R_a in Eq. (8) is a function of "a", the closed form integral becomes a very complicated equation. However, if it is assumed that R_a is constant over some small interval of crack growth, and that the constant "m" in Eq. (1) = 3.0, ΔN, the number of cycles required to grow the crack a distance Δa, or $(a_i - a_{(i+1)})$ can be readily determined by

$$\Delta N = \int_{a_i}^{a_{i+1}} \frac{da}{C(\sqrt{a}F_y)^3} = \frac{2(\frac{1}{\sqrt{a_i}} - \frac{1}{\sqrt{a_{(i+1)}}})}{C(F_y)^3} \tag{9}$$

where F_y is defined as $\Delta K/\sqrt{a}$ as determined from Eq. (8). This calculation can be repeated for successive intervals of Δa using the average value of R_a for that specific interval. By summing the values of ΔN for all intervals of "a" from a_o to the critical "a", the total fatigue life can be determined. The results of this calculation will converge to the value that would be obtained by a closed form integration as the value of Δa approaches zero. A computational scheme, which can easily be accomplished on a desk top or personal computer, is outlined as follows.

1. Using Eq. (2) and an iteration process to minimize σ_{tR}, determine the value of R at which $\sigma_{tR} = 0$. This is RZ. Also, in the same process, calculate σ_{tR} at R = RI, (σ_{tRI}) (this will be a negative number).

2. If $\sigma_{tRI} > -0.4$, then SB = σ_{tRI}.

3. If $\sigma_{tRI} < -0.4$, then calculate SB from Eq. (3).

4. Define Δa as the cylinder wall thickness divided by some number of intervals.

5. Select an initial crack depth, a_o, and define the first value of R as RI + a_o + $\Delta a/2$.

6. Calculate ΔK from Eq. (8) using the value of R determined above as R_a. A value of (SF) of 0.7 is suggested and the value of F to be used will be discussed later.

7. Calculate $a = R_a - RI$ and $F_y = \Delta K/\sqrt{a}$.

8. Calculate $a_i = a - \Delta a/2$ and $a_{(i+1)} = a + \Delta a/2$.

9. Calculate ΔN from Eq. (9). For values of "m" other than 3.0 a different form of Eq. (9) is required.

10. Let $R_a = R_a + \Delta a$ and repeat steps 6 and 7, summing ΔN values.

11. Continue incrementing R_a by Δa and summing ΔN's until ΔK exceeds KC for the material. KC may be the plane-strain fracture toughness, K_{Ic}, or a value somewhat larger. Actually, K_{max} and not ΔK should be used for this comparison, but since the crack growth rate is so large at this point, these details have very little effect on the total life calculated.

12. The value of $\sum \Delta N$ determined above is the calculated fatigue life.

NUMERICAL RESULTS

A computer program to perform the above calculations, has been written in Basic for use on an "Apple" computer. A copy of the program is given in the Appendix.

In order to determine the accuracy with which the proposed method can predict fatigue lives, particularly the effects of diameter ratio and autofrettage, a comparison is made with the experimental results of Davidson, et al (ref 16). This is shown in Figure 2. This reference was selected for comparison because it includes the widest range of the variables of interest. The fatigue lives for both autofrettaged and non-autofrettaged cylinders were calculated using the following constants and material properties.

SY = 160 Ksi	SF = 0.7
KC = 200 $Ksi\sqrt{in}$	F = 0.3
a_o = 0.005 in.	C = 3.4×10^{-10}
	m = 3.0

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[16] Davidson, T. E., Eisenstadt, R., and Reiner, A. N., "Fatigue Characteristics of Open-End, Thick-Walled Cylinders Under Cyclic Internal Pressure," ASME Paper No. 62-WA-164, J. Basic Engineering, Series D, Vol. 85, 1963, p. 555.

The crack depth interval over which ΔK is assumed to be constant is defined as the wall thickness divided by some number, M. To test the convergence of the numerical integration technique, several cases were run with M varying from 10 to 100. A plot of these results indicates that the value of life at $M = 40$ is less than 0.5 percent less than the value at $M = 100$. Therefore, $M = 40$ was used for the calculations shown in Figure 2.

The selection of the values $a_o = 0.005$ and $F = 0.3$ is somewhat arbitrary and was admittedly made to obtain agreement with the experimental results. However, the value of a_o is a reasonable estimate of the maximum material defect size that would be expected in the material used. The arbitrary selection of these values would not influence the agreement seen for the slopes of the curves or for the trend of diameter ratio effect on the autofrettaged cylinders. The selection of F does influence the difference between the autofrettaged and non-autofrettaged cylinders. However, for reasonable changes in the value of F from 0.1 to 0.5, the resulting change in stress at a given calculated life is only about 15 percent for the 2.0 diameter ratio, and less for smaller diameter ratios. This is very much less than the difference between the autofrettaged and non-autofrettaged results.

The results of the proposed method were also compared with the results of Throop and Fujczak (ref 17) which are the only known experimental results which systematically studied the effects of crack shape and extent of autofrettage (overstrain), and obtained fatigue crack growth data. This work

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[17] Throop, J. F. and Fujczak, R. R., "Strain Behavior of Pressurized Cracked Thick-Walled Cylinders," Experimental Mechanics, Vol. 22, August 1982, pp. 277-286.

utilized fairly deeply notched cylinders which forced the fatigue cracks to grow with specific crack shapes. For crack shape factors which would be appropriate for these specific shapes, according to the various references previously discussed, the calculated crack growth rates are significantly greater than those measured. This was noted by Parker, et al (ref 13) and others, and some possible explanations were discussed.

In order to obtain better agreement, shape factors for the three crack shapes used by Throop and Fujczak were obtained by using the proposed computer program and adjusting the shape factors until the calculated fatigue lives for the non-autofrettaged cylinders agreed with the experimental fatigue lives. This procedure gave shape factors of 0.65 for the long crack, 0.52 for the semi-elliptical crack, and 0.37 for the semi-circular crack. These values were used to calculate the crack growth curves for the autofrettaged cylinders and the results are shown in Figure 3 along with the experimental results of Throop and Fujczak. The proposed life prediction method generally underestimates the life of the notched cylinders by 10 to 30 percent. This is considered good agreement considering the inherent variability of this type of data. For the 60 percent overstrain, semi-circular notch specimen, the prediction slightly overestimated the life. However, there is reason to suspect that this experimental result may be lower than expected due to an additional defect in the cylinder.

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[13]Parker, A. P., Underwood, J. H., Throop, J. F., and Andrasic, C. P., "Stress Intensity and Fatigue Crack Growth in a Pressurized, Autofrettaged Thick Cylinder," ASTM STP 791, pp. I-216-I-237.

CONCLUSIONS

The fatigue life of a thick-walled cylinder subjected to a constant amplitude, cyclic, internal pressure can be calculated with reasonable accuracy by means of a relatively simple, fracture mechanics based analysis. This analysis considers the effects of crack shape, autofrettage residual stresses, corrected for the Bauschinger effect, and the negative values of the minimum stress intensity factor produced by the residual stresses.

The computations required for this analysis can be performed on a microcomputer, and a "Basic" program for performing these computations is provided.

REFERENCES

1. Eisenstadt, R., Kendall, D. P., and Davidson, T. E., "A Comparison of Results of Axial-Tension, Rotating-Beam, and Pressurized Cylinder Fatigue Tests," Experimental Mechanics, Vol. 9, No. 6, June 1969, pp. 250-254.
2. Karl, E., "The Fatigue Strength of Thick-Walled Cylinders," Proceedings of the 9th AIRAPT High Pressure Conference, Elsevier, 1984.
3. Timoshenko, S., Strength of Materials, Part II, Van Nostrand, 1956, pp. 386-392.
4. Franklin, G. J. and Morrison, J. L. M., "Autofrettage of Cylinders: Prediction of Pressure/External Expansion Curves and Calculation of Residual Stresses," Institution of Mechanical Engineers, London, Vol. 174, 1960, p. 947.
5. Sidebottom, O. M., Chu, S. C., and Lamba, H. S., "Unloading of Thick-Walled Cylinders That Have Been Plastically Deformed," Experimental Mechanics, Vol. 16, No. 12, December 1976, pp. 454-460.
6. Milligan, R. V., Koo, W. H., and Davidson, T. E., "The Bauschinger Effect in a High Strength Steel," ASME Publication No. 65-MET-9, 1965.
7. Kendall, D. P., "The Effect of Material Removal on the Strength of Autofrettaged Cylinders," Watervliet Arsenal Technical Report WVT-7003, January 1970 (available from NTIS).
8. Chen, P. C. T., "Prediction of Residual Stresses in an Autofrettaged Cylinder," Proceedings of the 9th AIRAPT High Pressure Conference, Elsevier, 1984.

9. Bowie, O. L. and Freese, C. E., "Elastic Analysis for a Radial Crack in a Circular Ring," Engineering Fracture Mechanics, Vol. 4, No. 2, 1972, pp. 315-321.

10. Pu, S. L. and Hussain, M. A., "Stress Intensity Factors For Radial Cracks in a Partially Autofrettaged Thick-Walled Cylinder," ASTM, STP 791, 1983, pp. I-194-I-213.

11. Tan, C. L. and Fenner, R. T., "Stress Intensity Factors For Semi-Elliptical Cracks in Pressurized Cylinders Using Boundary Integral Equation Method," Int. Jour. of Fracture, Vol. 16, No. 3, 1980, pp. 233-245.

12. Atulri, N. and Kathiresan, K., "3-D Analysis of Surface Flaws in a Thick-Walled Reactor Pressure Vessel Using Displacement-Hybrid Finite Element Method," Nuclear Engineering and Design, Vol. 51, 1979, pp. 163-176.

13. Parker, A. P., Underwood, J. H., Throop, J. F., and Andrasic, C. P., "Stress Intensity and Fatigue Crack Growth in a Pressurized, Autofrettaged Thick Cylinder," ASTM STP 791, 1983, pp, I-216-I-237.

14. Underwood, J. H. and Throop, J. F., "Surface Crack K-Estimates and Fatigue Life Calculations in Cannon Tubes," ASTM STP 687, 1979, pp. 195-210.

15. Newman, J. C. and Raju, I. S., "Analysis of Surface Cracks in Finite Plates Under Tension and Bending Loads," NASA, TN1578, 1979.

16. Davidson, T. E., Eisenstadt, R., and Reiner, A. N., "Fatigue Characteristics of Open-End, Thick-Walled Cylinders Under Cyclic Internal Pressure," ASME Paper No. 62-WA-164, J. Basic Engineering, Series D, Vol. 85, 1963, p. 555.

17. Throop, J. F. and Fujczak, R. R., "Strain Behavior of Pressurized Cracked Thick-Walled Cylinders," Experimental Mechanics, Vol. 22, No. 8, August 1982, pp. 277-286.

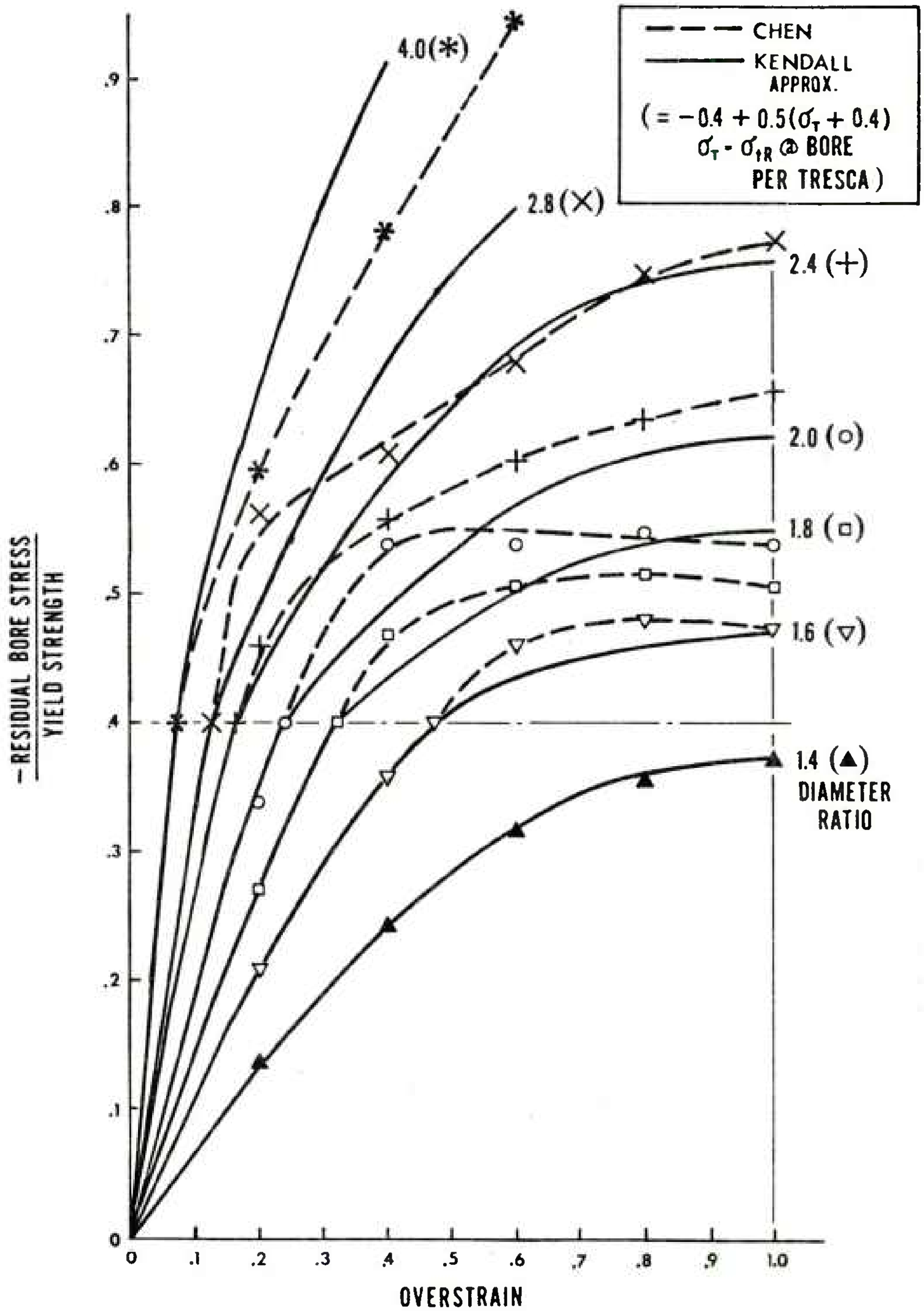
CHEN
KENDALL APPROX.
(= −0.4 + 0.5(σ_T + 0.4) σ_T - σ_tR @ BORE PER TRESCA)
4.0(✱)
2.8(×)
2.4 (+)
2.0 (○)
1.8 (□)
1.6 (▽)
1.4 (▲) DIAMETER RATIO
−RESIDUAL BORE STRESS / YIELD STRENGTH
OVERSTRAIN

Figure 1

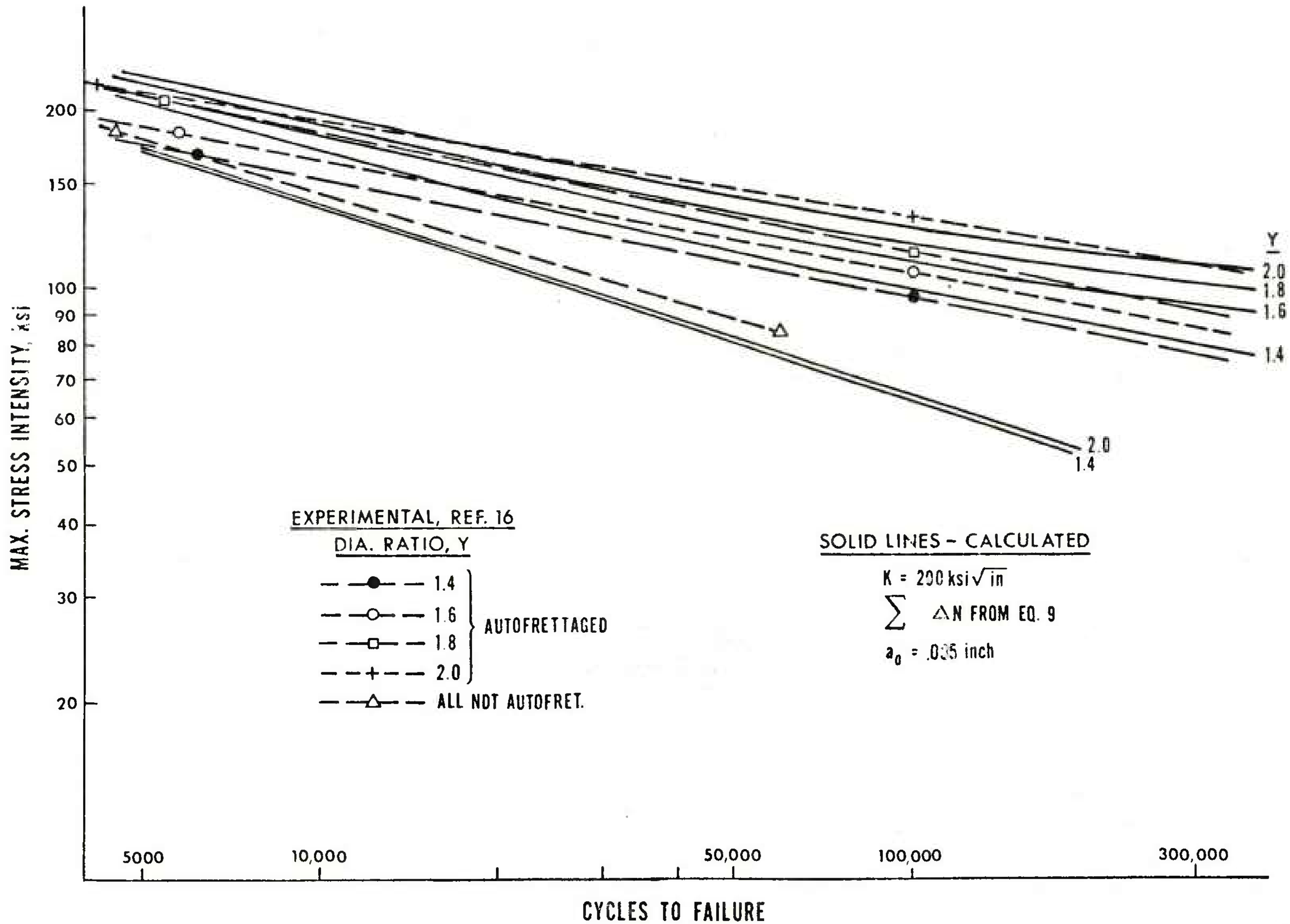
MAX. STRESS INTENSITY, ksi
200
150
100
90
80
70
60
50
40
30
20
5000
10,000
50,000
100,000
300,000
CYCLES TO FAILURE
Y
2.0
1.8
1.6
1.4
2.0
1.4
EXPERIMENTAL, REF. 16
DIA. RATIO, Y
1.4
1.6
1.8
2.0
AUTOFRETTAGED
ALL NOT AUTOFRET.
SOLID LINES - CALCULATED
K = 200 ksi√in
∑ ΔN FROM EQ. 9
a0 = .005 inch

Figure 2

20 INCH LONG NOTCH
SF = 0.65

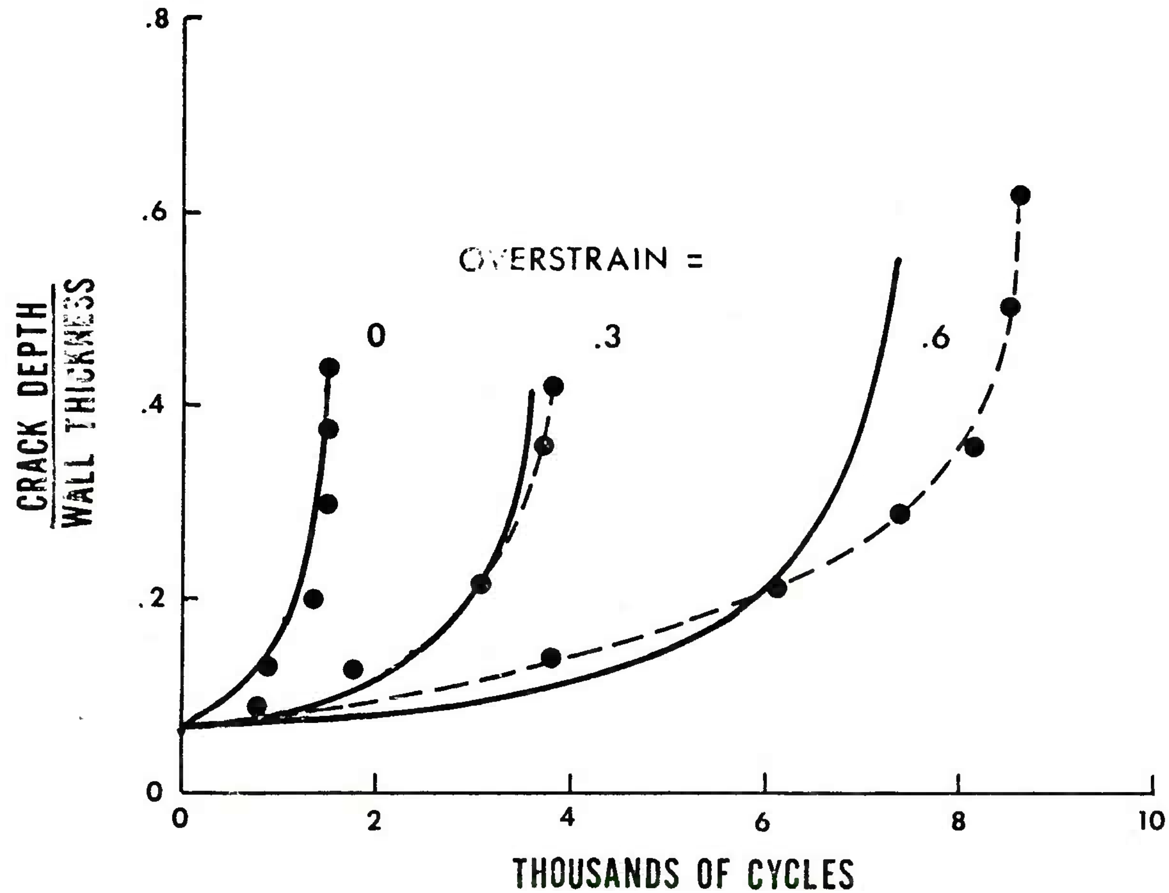

Figure 3A

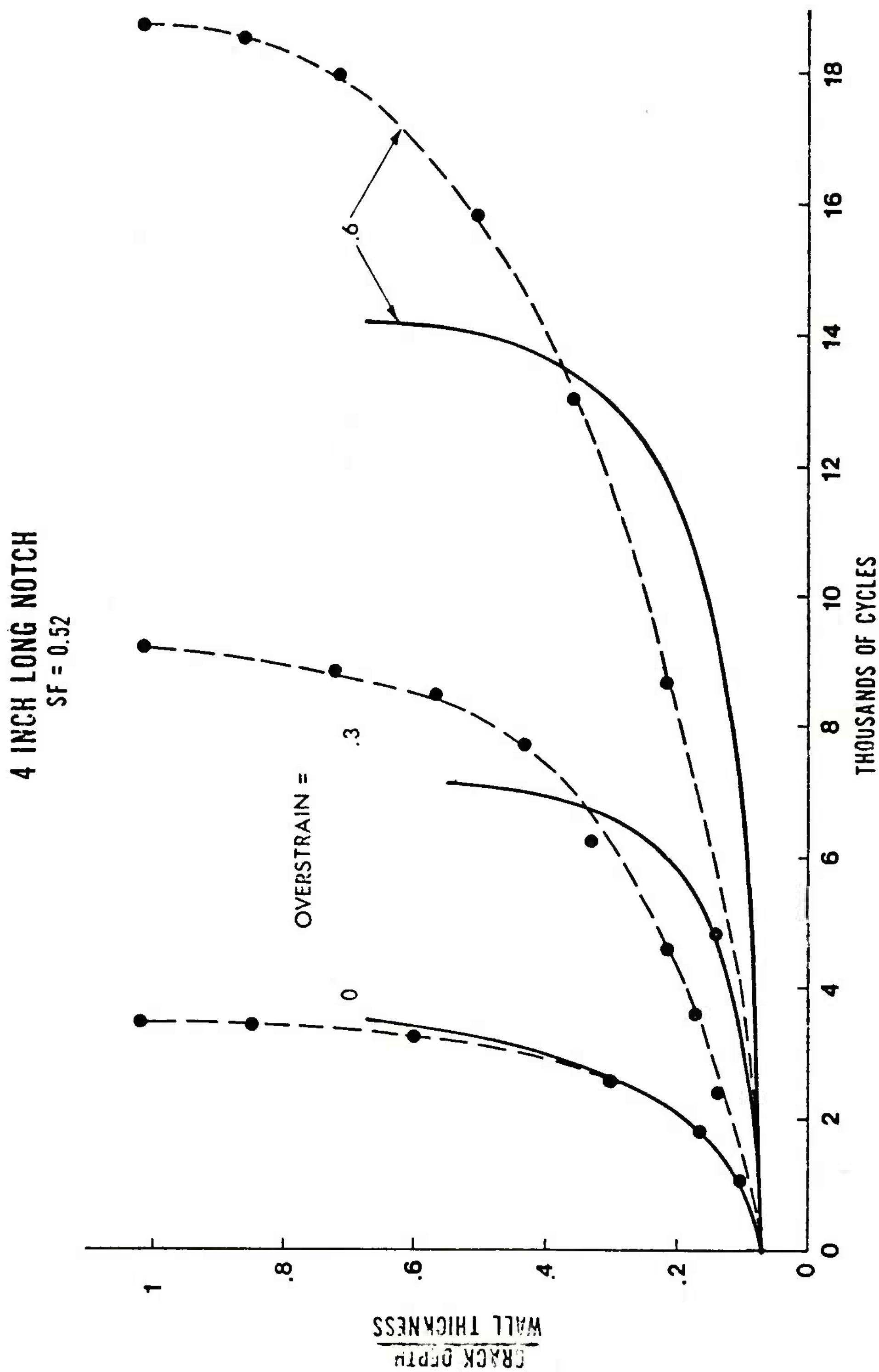
4 INCH LONG NOTCH
SF = 0.52
OVERSTRAIN = 0 .3 .6
CRACK DEPTH / WALL THICKNESS
0 .2 .4 .6 .8 1
0 2 4 6 8 10 12 14 16 18
THOUSANDS OF CYCLES

Figure 3B

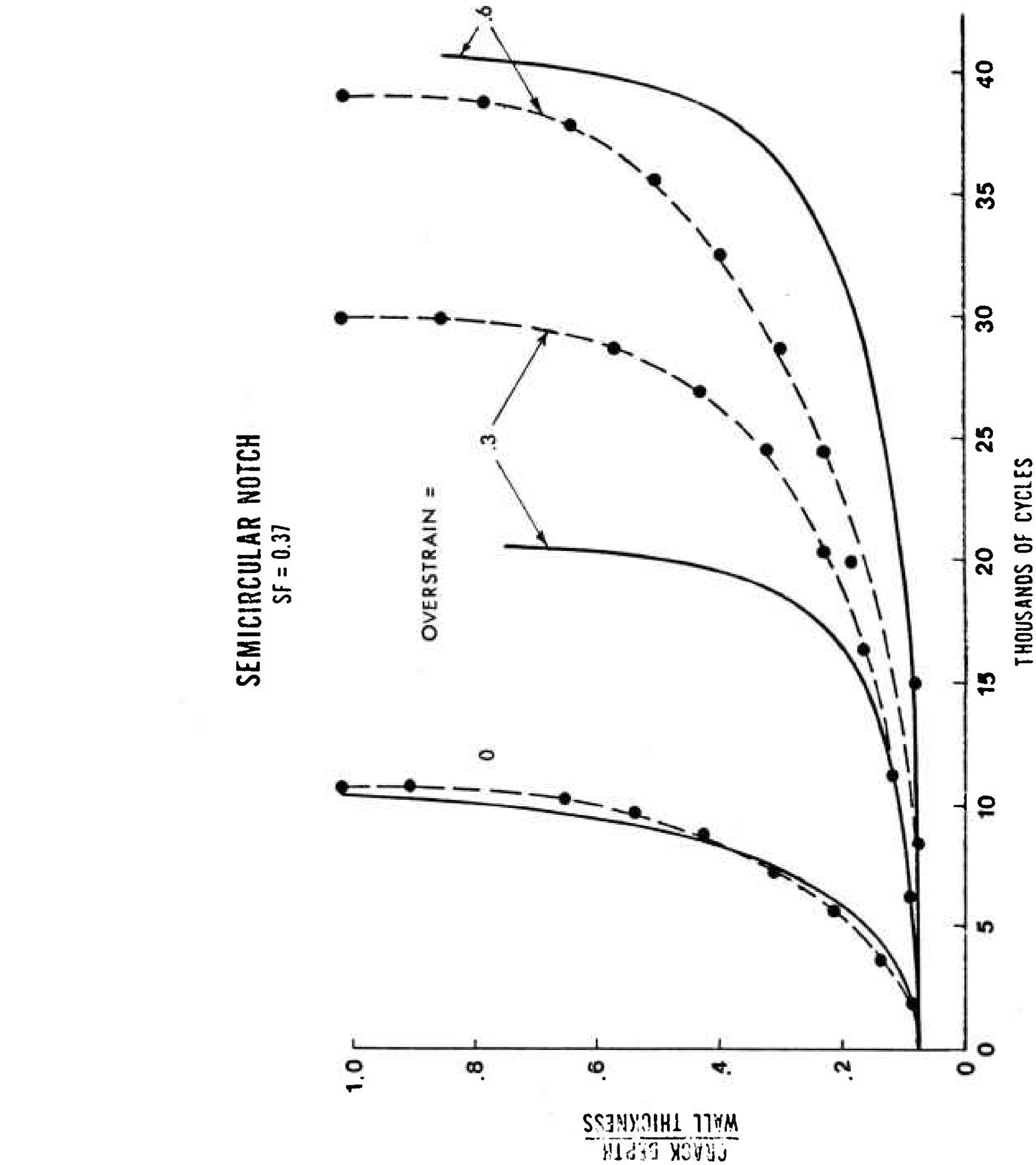
SEMICIRCULAR NOTCH
SF = 0.37
OVERSTRAIN =
0
.3
.6
CRACK DEPTH
WALL THICKNESS
1.0
.8
.6
.4
.2
0
0
5
10
15
20
25
30
35
40
THOUSANDS OF CYCLES

Figure 3C

APPENDIX

BASIC ("APPLESOFT") Program to Calculate Fatigue Life

of a Pressurized Cylinder

```
10  PRINT "SELECT REENTRY POINT"
11  PRINT "1=DIAMETER RATIO"
12  PRINT "2=YIELD STRENGTH"
13  PRINT "3=OVERSTRAIN"
14  PRINT "4=INITIAL CRACK DEPTH"
15  PRINT "5=INNER RADIUS"
16  PRINT "6=PRESSURE"
17  PRINT "7=END"
20  INPUT RE
23  PRINT "IF VALUE OK TYPE 0 OR SPACE, IF NOT ENTER NEW VALUE"
50  PRINT "DIAMETER RATIO = ";W
51  INPUT X: IF X THEN W = X
70  PRINT "YIELD STRENGTH IN KSI = ";SY
71  INPUT X: IF X THEN SY = X
80  PRINT "INNER RADIUS IN INCHES = ";RI
81  INPUT X: IF X THEN RI = X
100  INPUT "OVERSTRAIN RATIO = ";OS
110 RZ = 1
120 WE = W / ((W - 1) * OS + 1)
130 WP = W / WE
140 LWP =  LOG (WP)
150 R = (WP + 1) / 2
160 R5 = (WE ^ 2 + 1) / (2 * WE ^ 2)
170 R6 = (1 + (W / R) ^ 2) / (W ^ 2 - 1)
175 R7 = (WE ^ 2 - 1) / (2 * WE ^ 2) + LWP
180 LR =  LOG (R)
190 RS = R5 - (LWP - LR) - R6 * R7
210  IF R = 1 GOTO 300
220 RX =  ABS (RS)
230  IF RX < .00001 THEN  GOTO 240
235 R = R - (R * .3 * RS)
237  GOTO 160
240 RZ = R
279  PRINT
290 R = 1: GOTO 160
300 SB = RS
302  IF SB >  - 0.4 GOTO 305
304 SB =  - 0.4 + 0.5 * (SB + 0.4)
305  PRINT "RES BORE S/SY   R/RI AT RES S=0"
310  PRINT SB,RZ
311  PRINT
312 SB = SB * SY
313 RZ = RZ * RI
314 R2 = RI * W
```

```
320  PRINT "INITIAL CRACK DEPTH = ";AO
321  INPUT X: IF X THEN AO = X
335  INPUT "TO NOT PRINT K VS R TYPE 1";PK
340  PRINT "NO. OF INTEGRATION INTERVALS = ";M
341  INPUT X: IF X THEN M = X
350  PRINT "CRITICAL K IN KSI IN^1/2 = ";KC
351  INPUT X: IF X THEN KC = X
379  PRINT "PRESSURE IN KSI = ";P
380  INPUT X: IF X THEN P = X
381  PRINT "SHAPE FACTOR = ";SF
382  INPUT X: IF X THEN SF = X
385  PRINT "RESIDUAL K REDUCTION FACTOR = ";FK
386  INPUT X: IF X THEN FK = X
390 SA = (2 * P * W ^ 2) / (W ^ 2 - 1)
395 TN = 0
400 DR = RI * (W - 1) / M
406 R = RI + (DR / 2) + AO
410  IF RZ = RI THEN RZ = RZ + 1
415  PRINT  CHR$ (9)"80N"
416  IF PK = 1 GOTO 418
417  PRINT "CRACK DEPTH    K PRESSURE          K RESIDUAL         K TOTAL
       N CYCLES"
418 XC = (2.24 * P * W ^ 2) / (W ^ 2 - 1)
420 XA = 1.12 - .68 * (R - RI) / (RZ - RI)
430 XB = 3.142 * (R - RI)
440 KP = XC * ( SQR (XB)) * SF
442 KR = SB * XA * ( SQR (XB)) * SF
444 K = KP + KR * (1 - FK)
450 FY = SF * 1.7725 * (XC + SB * XA * (1 - FK))
451 A = R - RI
452 A1 = A - DR / 2:A2 = A + DR / 2
453 DN = ((1 /  SQR (A1) - 1 /  SQR (A2)) * 10 ^ 10) / (1.7 * FY ^ 3)
460 TN = TN + DN
465  IF PK = 1 GOTO 475
466  PRINT A,KP,KR,K,TN
475 R = R + DR
480  IF K > KC GOTO 495
485  IF R > R2 GOTO 495
490  GOTO 420
495  PRINT  CHR$ (9)"I"
499  PRINT : PRINT "N CYCLES       BORE STRESS INT"
500  PRINT TN,SA: PRINT
505  PRINT "REENTRY POINT = (7 TO END)";RE
506  INPUT X: IF X THEN RE = X
510  ON RE GOTO 50,70,100,320,80,379,650
650  END
```

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